ELEN 4810 Final Exam

Monday, December 19, 2022, 1:10-4:00 PM. Two sheets of handwritten notes are allowed. No electronics of any kind are allowed. Please record your answers in the exam booklet. Raise your hand if you need additional scratch paper.

There are a total of 4 questions. Good luck!

Name:

Uni:

1. Discrete Fourier Transform and Fast Fourier Transform. Consider two discrete time signals $u[n]$ and $v[n]$, which satisfy

$$\begin{array}{lll} u[n] \neq 0, & n = 0, 4, 8, 12, \ldots, 60, & u[n] = 0 \text{ else}, \\ v[n] \neq 0, & n = 0, 8, 16, 24, \ldots, 56, & v[n] = 0 \text{ else}. \end{array}$$

Set $y = u * v$. Please answer the following questions:

Part A. Suppose we compute y via the Discrete Fourier Transform, via

$$\mathrm{DFT}_N^{-1}\left(\mathrm{DFT}_N(u)[k] \, \mathrm{DFT}_N(v)[k] \right).$$

For what choices of N does this operation correctly compute y?

Part B. In this part, we use the structure of u and v to compute $y[n]$ more efficiently (similar to the Fast Fourier Transform). Let $\bar{u}$ and $\bar{v}$ be downsampled versions of u and v:

$$\begin{aligned} \bar{u} &= u \downarrow 4, \\ \bar{v} &= v \downarrow 8. \end{aligned}$$

Let

$$\begin{aligned} \bar{U}[k] &= \mathrm{DFT}_{32}\Big\{ \bar{u} \Big\}[k], \\ \bar{V}[k] &= \mathrm{DFT}_{16}\Big\{ \bar{v} \Big\}[k], \\ Y[k] &= \mathrm{DFT}_{128}\Big\{ y \Big\}[k]. \end{aligned}$$

Please give an expression for $Y[k]$ in terms of $\bar{U}$ and $\bar{V}$.

Answer to Problem 1:

2. Z **Transform.** Consider the following rational transfer function $H(z)$:

$$H(z) = \frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}.$$

Part a. What are the poles and zeros of H?

Part b. Assuming the system is *causal*, please specify the region of convergence (ROC) and the impulse response $h[n]$.

Part c. Assuming the system is *stable*, please specify the region of convergence (ROC) and the impulse response $h[n]$.

Part d. Which of the following best describes the system?

LOW PASS   BAND PASS   HIGH PASS   ALL PASS

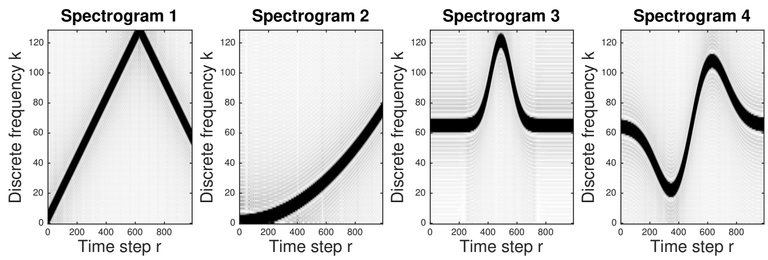
Answer to Problem 2:

3. Spectrograms. The following question has two parts.

Part (a). A signal $x[n]$ has the form

$$x[n] = \sin\Big(\alpha n + \beta \exp\big(-\gamma(n-\tau)^2\big)\Big),$$

for some scalars α, β, γ, τ.

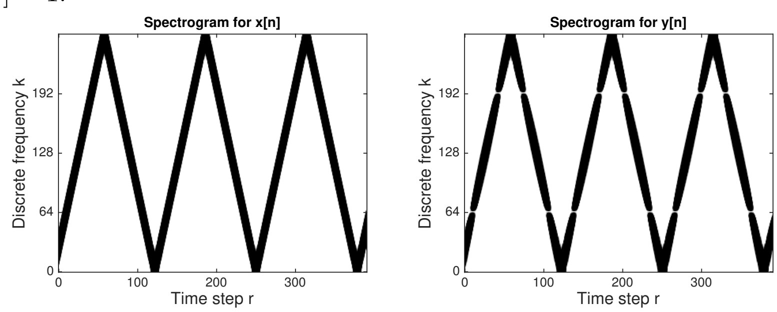


Which of the four figures above is the spectrogram of the signal? **For full credit, please justify your answer.**

Part (b). A linear chirp signal

$$x[n] = \cos\Big(\alpha n^2 + \beta\Big)$$

is passed through a canonical generalized linear phase system whose impulse response has length 5, and satisfies $h[0] = 1$.



Above are the spectrograms for $x[n]$ (left) and $y[n] = h * x[n]$ (right). Both spectrograms are generated with a Discrete Fourier Transform (DFT) of length $N = 512$.

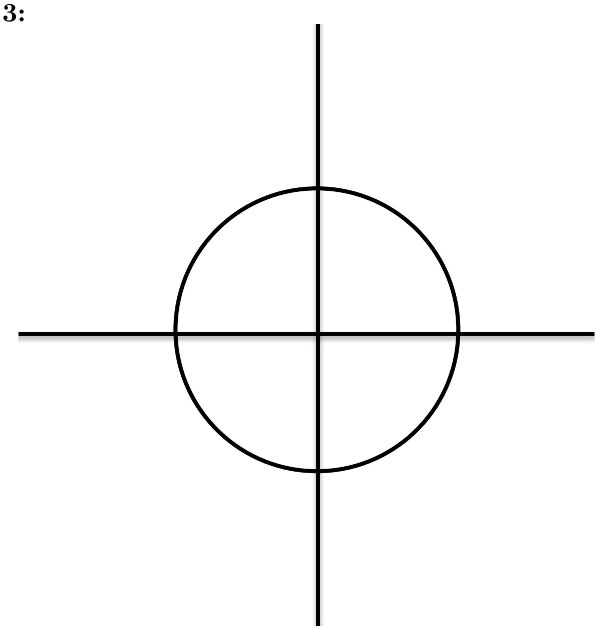
Please answer the following questions as accurately as possible, given the available information:

(b.i) What type of canonical generalized linear phase system is this?

(b.ii) What is the group delay $\text{grd}[H(e^{j\omega})]$?

(b.iii) Please sketch the pole-zero diagram of $H(z)$, using the axes on the next page. Please label any repeated poles and zeros with their multiplicity.

Answer to Problem 3:



Axis for Part b.iii

4. Filter Design by Windowing. In this problem, we design a low-pass filter by windowing. We set

$$H_{\text{target}}(e^{j\omega}) = \begin{cases} e^{-j\omega(L-1)/2} & |\omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \leq \pi. \end{cases}$$

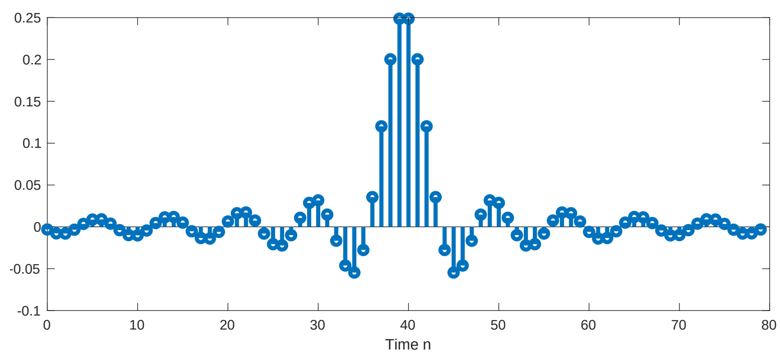
The corresponding time-domain target is

$$h_{\text{target}}[n] = \frac{\sin\left(\frac{\pi}{4}\left(n - \frac{L-1}{2}\right)\right)}{\pi\left(n - \frac{L-1}{2}\right)}$$

We use a rectangular window

$$w[n] = \begin{cases} 1 & 0 \leq n \leq L-1 \\ 0 & \text{else} \end{cases}$$

and set $h[n] = w[n]\, h_{\text{target}}[n]$. The impulse response $h[n]$ is plotted below, for $L = 80$:



Part A. Does the filter $h[n]$ have generalized linear phase? Why or why not?

Part B. In lecture, we discussed Kaiser windowing, which uses a different choice of $w[n]$. What is the main advantage of Kaiser windowing compared to the rectangular window used in part A?

Part C. What are the two main advantages of design by L^∞ optimization, compared to design by windowing?

Part D. Let $h[n]$ be our designed impulse response, $H(z)$ its Z-transform, and let $\zeta_1, \ldots, \zeta_M$ denote the zeros of $H(z)$.

Suppose we generate a new filter by setting $h_{\text{new}}[n] = (-1)^n h[n]$. **Please give an expression for the zeros $\zeta_1', \ldots, \zeta_M'$ of $H_{\text{new}}(z)$ in terms of the zeros $\zeta_1, \ldots, \zeta_M$.**

Part E. Which of the following best characterizes the filter $h_{\text{new}}[n]$? Why?

LOW PASS BAND PASS HIGH PASS ALL PASS

4. Answer to Problem 4: